INDIAN SCHOOL SOHAR No. of pages: 5 + 1 graph

TERM I EXAMINATION (2023-24)

MATHEMATICS (CODE -041)

CLASS: XII **MAX. MARKS: 80**

DATE: 26/9/23 **TIME: 3 Hours**

General Instructions:

1. This question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

SECTION – A
Multiple Choice Questions
(Each question carries 1 mark)

		MARKS
1.	If a relation R on the set {5,6,7} be defined by R ={(5,6)}, then R is a) Reflexive b) Transitive c) Symmetric d) None of these	1
2.	The function f: $[0, \infty) \to R$ given by $f(x) = \frac{x}{x+5}$ is: (a) f is both one-one and onto (b) f is one-one but not onto (c) f is onto but not one-one (d) neither one-one nor onto	1
3.	The value of $\sin[\,2\cot^{-1}(\frac{-5}{12})\,]$ is equal to a) $\frac{5}{12}$ b) $\frac{-5}{12}$ c) $\frac{169}{120}$ d) $\frac{-120}{169}$	1
4.	The value of $\cos^{-1}(-1) + \sin^{-1}(1)$ a) π b) $\frac{3\pi}{2}$ c) $\frac{\pi}{2}$ d) $\frac{-3\pi}{2}$	1
5.	If P and Q are two matrices of order 2 x p and 2 x q respectively, and p=q , then the order of the matric (P-3Q) is (a) q x2 (b)2 x 2 (c)p x q (d)2 x q	1
6.	If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then x equal to a) 3 b)± 6 c) ± 3 d) $\frac{1}{6}$	1
7.	The solution of set of inequality 3x+5y < 4 is a) an open half-plane not containing the origin b) an open half-plane containing the origin c) the whole xy-plane not containing the line 3x+5y=4 d) a closed half plane containing the origin	1

8.	Let $f(x) = \begin{cases} x + a \text{ if } x \geq 1 \\ ax^2 + 1 \text{ if } x < 1 \end{cases}$ then f is differentiable at x = 1 if (a) a = ½ (b) a=0 (c) a=2 (d) a= 1	1
9	Evaluate $\int e^x \sec x(\sec x + \sin x)dx$ a) e^x secx + c b) $e^x sec^2$x +c c) e^x tanx + c d) e^x secx tanx + c	1
10	The absolute maximum value of y = $x^3 - 3x + 2$ in $0 \leq x \leq 2$ is a)6 b) 4 c) 2 d) 0	1
11	Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer. a) R is reflexive and symmetric but not transitive. b) R is reflexive and transitive but not symmetric. c) R is symmetric and transitive but not reflexive. d) R is an equivalence relation.	1
12	Evaluate : $\sin\{\frac{\pi}{3} - \sin^{-1}((-\frac{1}{2}))\}$ a) 0 b) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) 1	1
13	If $A = \begin{bmatrix} k & 0 \\ 1 & 1 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then A^2=B is true for $a)\ k = -4$ b) $k = 4$ c) $k = 1$ d) for no value of k	1
14	If y = $\sqrt{sinx + \sqrt{sinx + \sqrt{sinx + \cdots..\infty}}}$ then $\frac{dy}{dx}$ is equal to a) $\frac{cosx}{2y-1}$ b) $\frac{cosx}{2y+1}$ c)0 d) None of these	1
15	If y = x^x then $\frac{d^2y}{dx^2}$ is equal to a) $x^x\left\{(1 + logx)^2 - \frac{1}{x}\right\}$ b) $x^x\left\{(1 + logx)^2 + \frac{1}{x}\right\}$ c) 0 d) $x^x\left\{(1 - logx)^2 + \frac{1}{x}\right\}$	1
16	The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which the area increases, when side is 10 cm is: a) 10 cm^2 /s b) $\sqrt{3}$ cm^2 /s c) 10 $\sqrt{3}$ cm^2 /s d) $\frac{10}{3}$$cm^2$ /s	1
17	$\int \frac{e^x(1+x)}{cos^2(x\, e^x)}$ dx is equal to a) $-\cot(e^x\, x) + c$ b) $\tan(e^x\, x) + c$ c) $\tan(e^x) + c$ d) $\cot(e^x) + c$	1
18	$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+cos2x}$ is equal to a) 1 b) 2 c) 3 d) 4	1

	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both (A) and (R) are true and (R) is the correct explanation of (A). (b) Both (A) and (R) are true but (R) is not the correct explanation of (A). (c) (A) is true but (R) is false. (d) (A) is false but (R) is true.	
19	**Assertion(A)**: Principal value of $\cos^{-1}(1)$ is π **Reason(R)**: Value of $\cos 0^0$ is 1	1
20	**Assertion(A)**: The function $[x(x-2)]^2$ is increasing in $(0,1) \cup (2, \infty)$ **Reason(R)**: $dy/dx = 0$ when $x = 0, 1, 2$	1
	SECTION B **(This section comprises of very short answer type questions (VSA) of 2 marks each)**	
21	Find $\int \frac{1}{cos^2x(1-tanx)^2}dx$	2
22	Evaluate : $\int_{-1}^{1} log_e\left(\frac{5-x}{5+x}\right)dx$	2
23	Show that the given function is one- one. $f: N \to N$ is defined by f(x) $=\begin{cases} x+1, if\ x\ is\ odd \\ x-1, if\ x\ is\ even \end{cases}$ **OR** Let f: R → R, such that f(x) = x^3. Check whether f is injective and surjective.	2
24	If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then find the value of k.	2
25	Find the value(s) of k so that the following function is continuous at $x = 0$ F(x) = $\begin{cases} \frac{1-\cos kx}{xsinx}, x \neq 0 \\ \frac{1}{2}, \ x = 0 \end{cases}$ **OR** If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, *then prove that* $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$	2
	SECTION C **(This section comprises of short answer type questions (SA) of 3 marks each)**	
26	Differentiate the following function w.r.t. x : y =$(sinx)^{tanx} + x^{cosx}$ **OR** If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$	3
27	Find the area of the region bounded by $x^2 = 4y$, y = 2, y = 4 and the y-axis in the first quadrant. **OR**	3

	Find the area of the region bounded by the curve $\frac{x^2}{16}+\frac{y^2}{9}=1$	
28	Make a rough sketch of the region $\{(x, y): 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$ and find the area of the region using integration.	3
29	Find the area of the triangle whose vertices are (3, 8), (– 4, 2) and (5, 1) **OR** Using cofactors of the elements of the second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.	3
30	Evaluate the following : $\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$	3
31	Solve the following linear programming problem graphically. Minimize Z = 5x+ 7y Subject to constraints; $2x + y \geq 8$ $x + 2y \geq 10$ $x, y \geq 0$	3
SECTION D **(This section comprises of long answer type questions (LA) of 5 marks each)**		
32	Let N be the set of all natural numbers and R be a relation on N x N defined by (a ,b) R (c ,d) ⇔ a d= b c for all (a, b) ,(c, d) ∈ N x N . Show that R is an equivalence relation on N x N. Also, find the equivalence class of (2, 6).	5
33	Using the matrix method, solve the following system of linear equations : $x - y + 2z = 7$ $3x + 4y - 5z = -5$ $2x - y + 3z = 12$	5
34	Evaluate: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ **OR** Evaluate: $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\,dx$	5
35	Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume. **OR** Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.	5

SECTION E **[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)**		
36	A rectangular hall is to be developed for a meeting of farmers in an agriculture college to conduct awareness amongst them about the new technique in cultivation. It is given that the floor has a fixed perimeter P.	
	i) If x and y represents the length and breadth of the rectangular region, then find the are function A in terms of x.	1
	ii) Find the critical point of the function A.	1
	iii) Use the First derivative test to find the length x and breadth y of rectangular hall that maximize its area. **OR** iii) Use the Second derivative test to find the length x and breadth y of rectangular hall that maximize its area.	2
37	A manufacturer produces three types of bolts, x, y and z which he sells in two markets. Annual sales (in Rs.) are indicated below :	

Markets	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

	If unit sales prices of x, y and z are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively, then answer the following questions using the concept of matrices.	
	i) Find the total revenue collected from the Market-I	1
	ii) Find the total revenue collected from the Market-II	1
	iii) If the unit costs of the above three commodities are Rs. 2.00, Rs.1.00 and 50 paise respectively, then find the gross profit from the First market **OR** iii) If the unit costs of the above three commodities are Rs. 2.00, Rs.1.00 and 50 paise respectively, then find the gross profit from the second market	2
38	The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation y = 6x - $\frac{1}{2}x^2$ where x is the number of days exposed to sunlight.	
	i) What is the number of days it will take for the plant to grow to the maximum height?	2
	ii) What is the maximum height of the plant?	2

---------------------THE END -----------------

MATHS TERM I-2023-24
SCORING KEY
STD XII

	(SECTION –A)	
1.	b) Transitive	**MARKS** 1
2.	b) f is one-one but not onto	1
3.	d) $; \sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$ $= \sin(2y)$ $= \frac{2\tan y}{1+\tan^2 y}$ $= \frac{2\left(\frac{-12}{5}\right)}{1+\left(\frac{-12}{5}\right)^2} = \frac{-\frac{24}{5}}{\frac{25+144}{25}} = \frac{-24}{5} \times \frac{25}{169} = \mathbf{\frac{-120}{169}}$	1
4.	b) $\frac{3\pi}{2}$	1
5.	(d)2 x q	1
6.	b)± 6	1
7.	b) an open half-plane containing the origin	1
8.	a) a = ½	1
9	c) e^x tanx + c	1
10	b)4	1
11	b) R is reflexive and transitive but not symmetric.	1
12	d) 1	1
13	d) for no value of k	1
14	a)	1
15	b) $x^x\left\{(1+logx)^2+\frac{1}{x}\right\}$	1
16	c) 10 $\sqrt{3}$ cm^2 /s	1
17	b) tan (e^x x) + c	1
18	a) 1	1
19	(d) (A) is false but (R) is true.	1

	Explanation: In case of Assertion: $\cos^{-1}(1) = y$ $\cos y = 1$ $\cos y = \cos 0°$ $[\because \cos 0° = 1]$ $\therefore$ $y = 0$ $\Rightarrow$ Principal value of $\cos^{-1}(1)$ is 0 Hence Assertion is in correct. Reason is correct.	
20	b) Both (A) and (R) are true but (R) is not the correct explanation of (A).	1
	SECTIONB	
21	$I = \int \frac{1}{cos^2 x\,(1-\tan x)^2}dx$ Put, $1 - \tan x = y$ So that, $-sec^2 x\, dx = dy$ $= \int \frac{-1\, dy}{y^2} = -\int y^{-2} dy$ $= +\frac{1}{y} + c = \frac{1}{1-\tan x} + c$	1 1
22	Let $f(x) = \log_e\left(\frac{2-x}{2+x}\right)$ We have, $f(-x) = \log_e\left(\frac{2+x}{2-x}\right) = -\log_e\left(\frac{2-x}{2+x}\right) = -f(x$ **So, $f(x)$ is an odd function.** $\therefore \int_{-1}^{1} \log_e\left(\frac{2-x}{2+x}\right)dx = 0.$	1 1
23	For one-one **Case I :** When x_1, x_2 are odd natural number. $\therefore$ $f(x_1) = f(x_2) \Rightarrow x_1+1 = x_2+1$ $\forall x_1, x_2 \in N$ $\Rightarrow x_1 = x_2$ *i.e.*, f is one-one. **Case II :** When x_1, x_2 are even natural number $\therefore$ $f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1$ $\Rightarrow x_1 = x_2$ *i.e.*, f is one-one. **Case III :** When x_1 is odd and x_2 is even natural number $f(x_1) = f(x_2) \Rightarrow x_1+1 = x_2-1$ $\Rightarrow x_2 - x_1 = 2$ which is never possible as the difference of odd and even number is always odd number. Hence in this case $f(x_1) \neq f(x_2)$ *i.e.*, f is one-one. OR F is one one and onto	1 1

24	$= \begin{bmatrix} (1)(3)+(2)(2) & (1)(1)+(2)(5) \\ (3)(3)+(4)(2) & (3)(1)+(4)(5) \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ 17 & 23 \end{bmatrix}$ low comparing LHS to RHS, we get $k = 17$	1 1
25	$\underset{x\to 0}{Lt} \frac{1-\cos kx}{x \sin x} = \underset{x\to 0}{Lt} \frac{2\sin^2\left(\frac{kx}{2}\right)}{x \sin x}$ $= \underset{x\to 0}{Lt} \frac{\frac{2\sin^2\left(\frac{kx}{2}\right)}{x^2}}{\frac{x\sin x}{x^2}}$ $= \frac{\underset{x\to 0}{Lt} \frac{2\sin^2\left(\frac{kx}{2}\right)}{\left(\frac{kx}{2}\right)^2} \times \left(\frac{k}{2}\right)^2}{\underset{x\to 0}{Lt} \frac{\sin x}{x}} = \frac{2\times 1\times \frac{k^2}{4}}{1}$ $\because f(x)$ *is continuous at* $x = 0$ $\therefore \underset{x\to 0}{Lt} f(x) = f(0)$ $\Rightarrow \frac{k^2}{2} = \frac{1}{2} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$ OR $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$ Let $sin^{-1}x = A$ and $sin^{-1}y = B$. Then x = sinA and y = sinB $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1 \Longrightarrow sinBcosA + sinAcosB = 1$ $\Longrightarrow \sin(A+B) = 1 \Longrightarrow A + B = sin^{-1}1 = \frac{\pi}{2}$ $\Longrightarrow sin^{-1}x + sin^{-1}y = \frac{\pi}{2}$ Differentiating w.r.to x, we obtain $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$	1+1
	SECTION C	
26	Let $u = x^{\cos x}, v = (\sin x)^{\tan x}$ Taking log on either side $\log u = \cos x.\log x, \log v = \tan x \log \sin x$ Differentiating w.r.t. x $\frac{1}{u}\frac{du}{dx} = \cos x.\frac{1}{x} + \log x(-\sin x), \frac{1}{v}\frac{dv}{dx} = \frac{\tan x.\cos x}{\sin x} + \log \sin x.\sec^2 x$ $\frac{du}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right), \frac{dv}{dx} = (\sin x)^{\tan x}(1 + \sec^2 x \log \sin x)$ $\therefore$ From (i) we get $\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right) + (\sin x)^{\tan x}[1 + \sec^2 x \log \sin x]$ OR	1+1+1

OR

Given $(\cos x)^y = (\sin y)^x$

Taking log on both sides

$\therefore \quad \log(\cos x)^y = \log(\sin y)^x$

$\Rightarrow \quad y\log(\cos x) = x\log(\sin y)$

Differentiating both sides w.r.t. x, we get

$$y\frac{1}{\cos x}.\frac{d}{dx}\cos x + \log(\cos x).\frac{dy}{dx} = x.\frac{1}{\sin y}.\frac{d}{dx}\sin y + \log\sin y.1$$

$$\Rightarrow \quad -y\frac{\sin x}{\cos x} + \log(\cos x).\frac{dy}{dx} = x\frac{\cos y}{\sin y}\frac{dy}{dx} + \log\sin y$$

$$\Rightarrow \quad -y\tan x + \log(\cos x)\frac{dy}{dx} = x\cot y\frac{dy}{dx} + \log\sin y$$

$$\Rightarrow \quad \log(\cos x).\frac{dy}{dx} - x\cot y\frac{dy}{dx} = \log\sin y + y\tan x$$

$$\Rightarrow \quad \frac{dy}{dx}[\log(\cos x) - x\cot y] = \log\sin y + y\tan x$$

$$\therefore \quad \frac{dy}{dx} = \frac{\log\sin y + y\tan x}{\log\cos x - x\cot y}$$

27

Area of ABCD $= \int_2^4 x\,dy$ 1

$$= \int_2^4 2\sqrt{y}dy$$

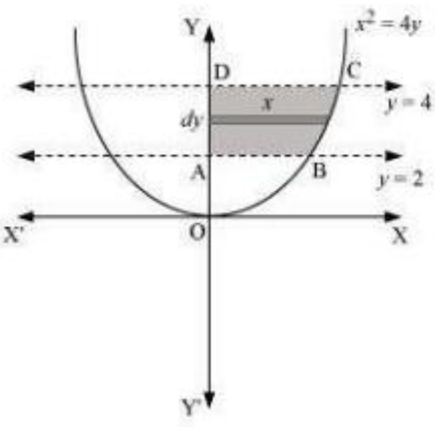
$$= 2\int_2^4 \sqrt{y}dy$$

$$= 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4$$ 1

$$= \frac{4}{3}\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right]$$ 1

$$= \frac{4}{3}\left[8 - 2\sqrt{2}\right]$$

$$= \left(\frac{32 - 8\sqrt{2}}{3}\right) \text{ units}$$



Area of OAB $= \int_0^4 y\,dx$

$$= \int_0^4 3\sqrt{1 - \frac{x^2}{16}}dx$$

$$= \frac{3}{4}\int_0^4 \sqrt{16 - x^2}dx$$

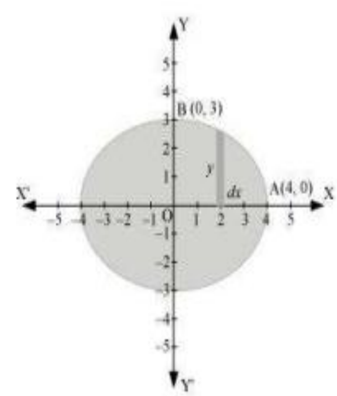
$$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$

$$= \frac{3}{4}\left[2\sqrt{16-16} + 8\sin^{-1}(1) - 0 - 8\sin^{-1}(0)\right]$$

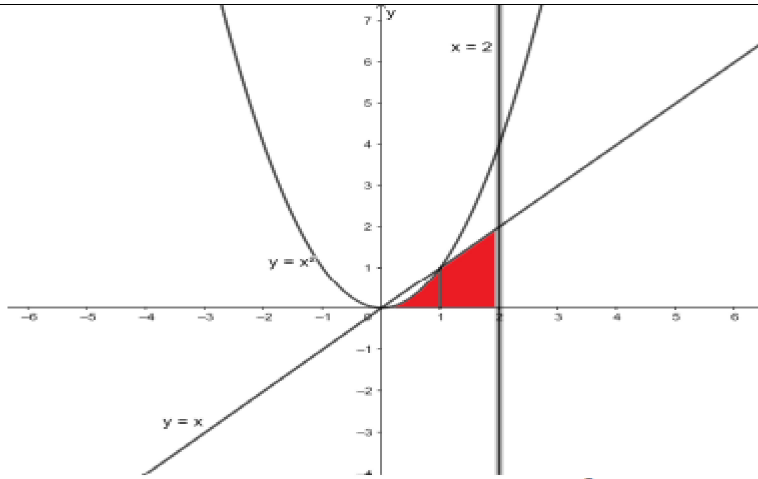
$$= \frac{3}{4}\left[\frac{8\pi}{2}\right]$$

$$= \frac{3}{4}[4\pi]$$

$$= 3\pi$$



Therefore, area bounded by the ellipse = 4 × 3π = 12π units

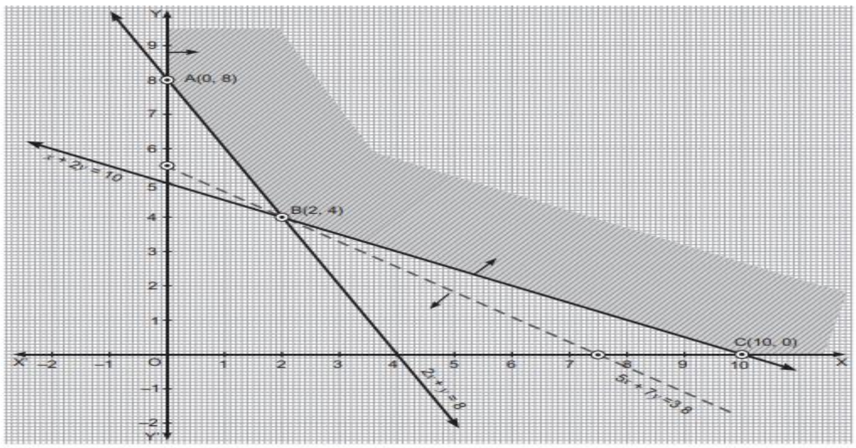
28	 The points of intersection of the parabola $y = x^2$ and the line $y = x$ are (0, 0) and (1, 1). Required Area = $\int_0^1 y_{parabola}\,dx + \int_1^2 y_{line}\,dx$ Required Area = $\int_0^1 x^2\,dx + \int_1^2 x\,dx$ $= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$	1 1 1
29	The area of triangle is given by $\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$ $= \frac{1}{2}\left[3(2-1) - 8(-4-5) + 1(-4-10)\right]$ $= \frac{1}{2}(3 + 72 - 14) = \frac{61}{2}$ OR $\therefore \Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = 2(7) + 0(7) + 1(-7) = 14 - 7 = 7$	1 1 1
30	Given, $\int_1^2 \frac{5x^2}{x^2 + 4x + 3}\,dx$ $= 5\int_1^2 \frac{(x^2 + 4x + 3) - (4x + 3)}{x^2 + 4x + 3}\,dx = 5\int_1^2 dx - 5\int_1^2 \frac{4x + 3}{x^2 + 4x + 3}\,dx$ $= 5[x]_1^2 - 5\int_1^2 \frac{4x + 8 - 5}{x^2 + 4x + 3}\,dx = 5 - 5\left[\int_1^2 \frac{2(2x + 4)}{x^2 + 4x + 3}\,dx - 5\int_1^2 \frac{dx}{x^2 + 4x + 3}\right]$ $= 5 - 10\int_1^2 \frac{2x + 4}{x^2 + 4x + 3}\,dx + 25\int_1^2 \frac{dx}{(x + 2)^2 - 1}$ $= 5 - \left[10 \log\lvert x^2 + 4x + 3\rvert - \frac{25}{2}\log\left\lvert\frac{x + 1}{x + 3}\right\rvert\right]_1^2$ $= 5 - \left[10 \log 15 - \frac{25}{2}\log\frac{3}{5} - 10\log 8 + \frac{25}{2}\log\frac{1}{2}\right] = 5 + 10\log\frac{8}{15} + \frac{25}{2}\log\frac{6}{5}$	1 1 1

31 | 1+1+1

Corner Point	z = 5x + 7y
A (0, 8)	56
B (2, 4)	38 ← Minimum
C (10, 0)	50

ıce feasible region is unbounded. Therefore we have to draw the graph of the inequality.

$$5x + 7y < 38 \qquad ...(vi)$$



he minimum value of z is 38 at (2, 4).

SECTION D

32 | 1, 2, 2

$\Rightarrow (a,b) R (a,b)$, according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$

Thus $(a,b) R (a,b),\ \forall (a,b) \in \mathbb{N}\times\mathbb{N}$.

So, R is reflexive relation on $\mathbb{N}\times\mathbb{N}$.

Let $(a,b),(c,d)$ be arbitrary elements of $\mathbb{N}\times\mathbb{N}$ such that $(a,b) R (c,d)$.

Then, $(a,b) R (c,d) \Rightarrow ad = bc \Rightarrow bc = ad$; (changing LHS and RHS)

$\Rightarrow cb = da$; (As $a,b,c,d \in \mathbb{N}$ and multiplication is commutative on $\mathbb{N}$)

$\Rightarrow (c,d) R (a,b)$; according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$

Thus $(a,b) R (c,d) \Rightarrow (c,d) R (a,b)$

So, R is symmetric relation on $\mathbb{N}\times\mathbb{N}$.

Let $(a,b),(c,d),(e,f)$ be arbitrary elements of $\mathbb{N}\times\mathbb{N}$ such that

$(a,b) R (c,d)$ and $(c,d) R (e,f)$.

Then $\left.\begin{matrix}(a,b) R (c,d) \Rightarrow ad = bc \\ (c,d) R (e,f) \Rightarrow cf = de\end{matrix}\right\} \Rightarrow (ad)(cf) = (bc)(de) \Rightarrow af = be$

$\Rightarrow (a,b) R (e,f)$; (according to the definition of the relation R on $\mathbb{N}\times\mathbb{N}$)

Thus $(a,b) R (c,d)$ and $(c,d) R (e,f) \Rightarrow (a,b) R (e,f)$

$$[(2,6)] = \{(x,y) \in \mathbb{N}\times\mathbb{N} : (x,y) R (2,6)\}$$
$$= \{(x,y) \in \mathbb{N}\times\mathbb{N} : 3x = y\}$$
$$= \{(x,3x) : x \in \mathbb{N}\} = \{(1,3),(2,6),(3,9),.........\}$$

33	Now, $\|A\| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix} = 1(12-5)+1(9+10)+2(-3-8)$ $= 7 + 19 - 22 = 4 \neq 0$ $\therefore \quad adjA = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$ $\Rightarrow \quad A^{-1} = \frac{1}{\|A\|} adj\, A = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$ $\because \quad AX = B$ $\Rightarrow \quad X = A^{-1}B$ $\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$ $\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix}$ $\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ Equating the corresponding elements, we get $x = 2,\ y = 1,\ z = 3$ OR	1 2.5 1.5
34	Let $\quad I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}$ $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}\, dx}{\sqrt{\cos x}+\sqrt{\sin x}}$ $= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}\, dx}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}$ $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}\, dx}{\sqrt{\sin x}+\sqrt{\cos x}}$ Adding (*i*) and (*ii*), $2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$ $2I = \int_{\pi/6}^{\pi/3} dx = [x]_{\pi/6}^{\pi/3}$ $\therefore \quad I = \frac{1}{2}\left[\frac{\pi}{3}-\frac{\pi}{6}\right] = \frac{1}{2}\left[\frac{2\pi-\pi}{6}\right]$ $I = \frac{\pi}{12}$	1 1 1 1 1

OR

Let $I = \int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\,dx = \int \frac{6x+7}{\sqrt{x^2-9x+20}}\,dx$

Now, Let $6x+7 = A.\frac{d}{dx}(x^2-9x+20)+B$

$6x+7 = A(2x-9)+B$

$\Rightarrow$ $6x+7 = 2Ax-9A+B$

Comparing the coefficient of x, we get

$2A = 6$ and $-9A+B=7$

$A = 3$ and $B = 34$

$\therefore$ $I = \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}\,dx$

$= 3\int \frac{(2x-9)\,dx}{\sqrt{x^2-9x+20}} + 34\int \frac{dx}{\sqrt{x^2-9x+20}}$

$I = 3I_1 + 34I_2$

where $I_1 = \int \frac{(2x-9)\,dx}{\sqrt{x^2-9x+20}}$ and $I_2 = \int \frac{dx}{\sqrt{x^2-9x+20}}$

$= 6\sqrt{x^2-9x+20} + 34\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right| + C$

---

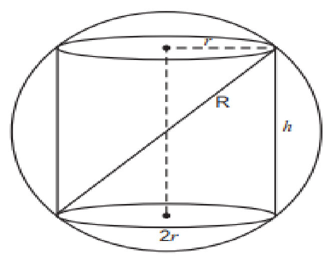
35 1

Let r, h be the radius and height of the cylinder inscribed in the sphere of radius R.

$\therefore$ Using Pythagoras theorem

$4r^2 + h^2 = 4R^2$

$\Rightarrow$ $r^2 = \frac{4R^2-h^2}{4}$ ... (*i*) 1



Volume of cylinder $= V = \pi r^2 h$

$\Rightarrow$ $V = \pi.h\left(\frac{4R^2-h^2}{4}\right) = \pi R^2 h - \frac{\pi}{4}h^3$

$\Rightarrow$ $\frac{dV}{dh} = \pi R^2 - \frac{3\pi}{4}h^2$ ... (*ii*) 1

For finding maximum volume

$\frac{dV}{dh} = 0 \Rightarrow \pi R^2 = \frac{3\pi}{4}h^2$

$\Rightarrow h = \frac{2}{\sqrt{3}}R$ 1

Differentiating (*ii*) again

$\frac{d^2V}{dh^2} = -\frac{6\pi}{4}h$

$\frac{d^2V}{dh^2}\Big|_{\left(h=\frac{2}{\sqrt{3}}R\right)} = -\frac{3\pi}{2}\left(\frac{2}{\sqrt{3}}R\right) = -\sqrt{3}R\pi < 0$ 1

Hence volume is maximum when $h = \frac{2}{\sqrt{3}}R$.

Maximum volume $= V\Big]_{h=\frac{2R}{\sqrt{3}}} = \pi h\left(\frac{4R^2-h^2}{4}\right)$

$V_{max} = \pi \times \frac{2R}{\sqrt{3}}\left(\frac{4R^2-\frac{4R^2}{3}}{4}\right)$

$= \frac{2\pi R}{\sqrt{3}}.\frac{2R^2}{3} = \frac{4\pi R^3}{3\sqrt{3}}$ cubic units.

OR

	Let r be the radius and h be the height of the cylinder of given surface s. Then, $s = \pi r^2 + 2\pi hr$ $h = \dfrac{s - \pi r^2}{2\pi r}$ ...(i) Then $v = \pi r^2 h = \pi r^2 \left[\dfrac{s - \pi r^2}{2\pi r}\right]$ [From eqn. (i)] $v = \dfrac{sr - \pi r^3}{2}$ $\dfrac{dv}{dr} = \dfrac{s - 3\pi r^2}{2}$ ...(ii) For maximum or minimum value, we have $\dfrac{dv}{dr} = 0$ $\Rightarrow \dfrac{s - 3\pi r^2}{2} = 0 \Rightarrow s = 3\pi r^2$ $\Rightarrow \pi r^2 + 2\pi rh = 3\pi r^2$ $\Rightarrow r = h$ Differentiating equation (ii) w.r.t. r, we get $\dfrac{d^2v}{dr^2} = -3\pi r < 0$	

SECTION E

36 i) A= xy= x (P-2x)/2 i) dA/dx=0; x =P/4 ii) dA/dx greater than 0 , x belongs to (0,P/4) dA/dx less than 0 , x belongs to (P/4,P/2) x= P/4 = y OR Second derivative = -2 (less than 0) So maxima Point= x=p/4 =y	1 1 2
37 Now, revenue = sale price × number of items sol $= \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$ $= \begin{bmatrix} 25000 + 3000 + 18000 \\ 15000 + 30000 + 8000 \end{bmatrix} = \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$ **1)**, 46000 2) 53000 $\therefore$ Total cost in each market is given by $AC = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$ $= \begin{bmatrix} 20000 + 2000 + 9000 \\ 12000 + 20000 + 4000 \end{bmatrix} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$ Now, Profit matrix = Revenue matrix − Cost matrix $= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix} = \begin{bmatrix} 15000 \\ 17000 \end{bmatrix}$ 3) 15000 OR 17000	1 1 2
38. 1) 6 **2)** 18	2 2

MATHS TERM I -2023-24

STD XII

1. This question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

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Sl.no	CHAPTER	MCQ1 MARK	CBQ 4 MARK	2 MARKS	3 MARKS	5 MARKS	Total 38(80 marks)
1	**Relation and Function**	**1+1+1**		**1***		**1**	**5(10 marks)**
2	**Inverse Trigonometry**	**1+1+1+1**					**4(4marks)**
3	**Matrices**	**1+1**	**1**	**1**			**4(8marks)**
4	**Determinants**	**1**			**1***	**1**	**3(9 marks)**
5	**Continuity and Differentiability**	**1+1+1**	**1**	**1***	**1***		**6(13marks)**
6	**Application of Derivatives**	**1+1+1+1**	**1**			**1***	**6(13 marks)**
7	**Integrals**	**1+1**		**1+1**	**1**	**1***	**6(14 marks)**
8	**Application of Integrals**				**1+1***		**2(6 marks)**
9	**Linear Programming**	**1**			**1**		**2(4marks)**

Each student 1 graph